Robust Regularization of the Boundary Integral Method for Noise Scattering

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Abstract

This work presents a newly developed tool for noise scattering based on a regularized form of the boundary integral method. The tool can be used to include the effects of solid surfaces on noise propagation. In addition to this, it can be used to compute the acoustic load on solid surfaces located in the vicinity of arbitrary noise sources. The tool is consists of two parts. In the first part, the incident acoustic field is computed using the Ffowcs Williams – Hawkings (FW-H) method. To this end, high-fidelity CFD/CAA simulations are used to provide the input data to the FW-H method. After this, the scattered acoustic field is computed using a regularized boundary integral method. By using a regularized form of the integral, all singularities of the integral are removed analytically, which in turn enables us to use a consistent quadrature rule over the entire solid surface. The focus of this talk will be on the steps taken to formulate a numerically robust regularization that works with complex meshes. For validation, both analytical solutions and numerical test cases are considered.

Method

The boundary integral method can be expressed in compact form as [1]

$$c(x)\hat{p}_{sc}(x) = \iint_S \hat{p}_{sc}(y)G_n(x;y) - \hat{p}_{sc,n}(y)G(x;y)dS. \tag{1}$$

Here, x, y denote the point where the pressure is computed and the point on the surface where the integral is being evaluated, respectively. The function $G(x;y)$ is the Green's function to the Helmholtz equation, $\hat{p}_{sc}$ is the Fourier transform of the scattered pressure, $\square_n$ denotes the surface-normal derivative, and $c(x)$ is a constant that depends on whether x is located on the surface or outside the surface. In the latter case, it is simply 4π, whereas in the former case, it depends on the surface geometry. Equation (1) holds for any field that satisfies the three-dimensional Helmholtz equation in the volume outside the surface S. In addition to this, it must satisfy the Sommerfeld radiation condition. Otherwise, Eq. (1) will contain an additional integral over the surface at infinity.

Our goal is to compute the scattered pressure on the surface using the normal derivative of the pressure as input. The normal derivative of the scattered pressure is computed based on the boundary condition

$$\frac{\partial(p_{in} + p_{sc})}{\partial n} = 0. \tag{2}$$

That is, we require that the normal derivative of the total (incident + scattered) pressure is 0 at the surface. The normal derivative of the incident pressure is computed using the FW-H method, and is therefore considered known. As a result, the normal derivative of the scattered pressure is also known from the boundary condition above.

To obtain the scattered pressure on the surface, the integral in Eq. (1) can be discretized over the solid surface. If Gauss quadrature is used, then the right hand side will simply be a weighted sum of the values of the

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integrand at each quadrature node on the surface. If the point x is then chosen to be one of the quadrature nodes, we obtain a relation between the pressure at this node, and the pressure and normal derivative of the pressure at all other nodes. By doing this for all points on the surface, we get a linear system of equations for the unknown scattered pressure at the nodes.

Unfortunately, the above procedure is complicated by the fact that the integral is singular as $y \rightarrow x$. This is because the Green's function to the Helmholtz equation is proportional to $1/|x-y|$. In addition to this, the constant $c(x)$ changes values depending on whether the surface is smooth or not. These difficulties can be handled by analytically integrating the Green's function in the vicinity of singularities and, e.g., always choosing the point x in the middle of a mesh element, for which $c(x) = 2\pi$. In this work, on the other hand, we follow the work of [1] and start by removing the singularities analytically. After this, the integral can be computed using the same quadrature rule for all points on the surface.

To remove the singularity, an auxiliary field $\psi(y_s; y)$ that satisfies the Helmholtz equation and the following two conditions is introduced.

$$\psi(y_s; y_s) = \hat{p}(y_s), \qquad \psi_n(y_s; y_s) = \hat{p}_n(y_s). \tag{3}$$

Here, y_s is some point on the surface S. Since the auxiliary field satisfies the Helmholtz equation, it also satisfies Eq. (1). Therefore, we may subtract Eq. (1) with $p_{sc}(y) \rightarrow \psi(x; y)$ (i.e., for $\psi(y_s, y)$ with y_s selected to be x) from Eq. (1) to obtain

$$0 = \iint_S \left(\hat{p}_{sc}(y) - \psi(x;y)\right) G_n(x;y) - \left(\hat{p}_{sc,n}(y) - \psi_n(x;y)\right) G(x;y)\, dS. \tag{4}$$

Note that, in the above equation, the terms inside the parentheses go to zero as $y \rightarrow x$. Therefore, the integrand will be bounded if the terms inside the parentheses go to zero at the same rate or faster than the Green's function that multiply the parentheses. If an auxiliary field that satisfies this condition together with Eq. (3) can be found, then the integral is regular and can thus be computed with standard quadrature methods. This will allow us to construct a linear system of equations for the unknown scattered pressure at the surface as explained earlier.

Outlook on talk at conference

The presentation will focus on how to select the auxiliary field such that a robust regularization is obtained. A set of validation cases based on analytical solutions and numerical test cases will also be presented to verify the correctness and robustness of the chosen regularization approach.

References

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